Understanding a Mediæval Algorithm : a Few Examples in Arab and Latin Geometrical Traditions of Measurement

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Taking into account written texts from Arab and Latin traditions of the geometry of measurement[1], our main purpose is to describe several elements of algorithms in order to analyze how their part of the explicitness and that of the tacitness could help the historian of mathematics to understand computations.

After the introduction where the context is briefly exposed, we will focus on two different classical examples of the geometry of measurement. The first one is a series of problems on rectangle where additive relations on area, length and width are done, and it is necessary to find both length and width. The second problem is a sharing land between heirs.

Introduction.

(1) How can we understand algorithms in mediæval texts which are often, at first sight, obscure for a present-day reader ? (2) And are we able to describe elements which guarantee the correctness of those algorithms? And especially, how the author of this kind of algorithms, and after, readers and finally historians, are sure that the solution given, following step by step the algorithm, answers to the problem?

Here, the notion of *transparency* of an algorithm proposed by K. Chemla [1, p.260] could appear as a key concept. Unfortunately, reading strictly algorithms given in mediæval texts of measurement, almost all, if not all, are not transparent. Indeed, at least, each step give us the number established by computation but it does not make the meaning of the computations and this of the magnitudes explicit. We are in presence of tacit knowledge, at least in formulation. But, what kind of tacit knowledge is it exactly? Is it, for example, a tacit formulation wanted by the authors themselves to transfer their knowledge as clear as possible or anything else?

Thus, several other fundamental questions can be formulated by historians of mathematics : 1) How can we understand and interpret numbers in algorithms? 2) How and how far are we legitimate to reconstruct steps in algorithms which seem lacking? and the last but not least 3) What kind of proof of the correctness of the algorithm could we establish?

These three questions strictly depend on what is tacit and explicit in mathematical texts. In most of cases, only the mathematical tradition (here the geometry of measurement) and our knowledge of the cultural context (here, Islamic mathematics and its appropriation by Latin Europe) can help us to overcome difficulties.

[1]The geometry of measurement is called in Arabic classifications of sciences (from the ninth century) and in geometrical texts themselves : *ᶜilm al-misāḥa* or *sināᶜat at-taksīr*. In the Latin world, from the 12th century, this kind of texts belongs to the corpus of *Practica geometriæ*.

That is we want to show here.

Series of Problems : Let $A + \alpha w + \beta L$ and $L - w$ be given, with $\alpha, \beta \in \{-2, -1, 0, 1, 2\}$. L, w ? (If A area, L length and w width of a rectangle)

We focus on two main texts dealing with this kind of problems.

The first one is the *risālat fī t-taksīr* written by Ibn ᶜAbdūn during the tenth century. This text is only known by one manuscript kept in the French National Library. It is interesting to add that, as far as we know, this copy comes from the Umarian Library of Segou (Mali) [3].

The second one is the *Liber mensurationum* which is an Arab-Latin translation probably made in the twelfth century by Gherardo Cremona in Toledo. The author is only known by a part of his name : Abū Bakr which is not sufficient to identify him. We know this text thanks to, at least, 5 copies held in Paris (for 3 of them) and in Cambridge and Dresden (for 2 of them)[5].

These two texts are "texts of procedures", that is to say : they are exclusively composed by series of problems all structured on statement and algorithm of resolution. Geometrical or arithmetical proves do not complete the text.

This type of problems is interesting for several reasons, and in particular because the 'tacitness' can be specify at different levels[2]. The first level is about the numbers used in the computations. Then, in order to understand the algorithm, we have to know precisely what each numerical value represent tacitly as magnitude. It is a necessary condition to write a mathematical analysis autorizing us to formulate tacit steps. Thus, the fundamental question is to know the reason why the author didn't write some steps: it can be wanted by himself or the text that we know can be corrupted. The third level is about the correctness of the algorithm. We have to render the tacit explicit, in particular giving a geometrical interpretation of the numerical problem[4]. The last level of tacitness we would like to enounce is linked to the organization of the series of problem. Indeed, we think that authors organize their series of problem to elaborate a pseudo-theory with all possible cases[3].

Sharing land between heirs : a socio-cultural problem borrowed by authors of mathematics. A case study in the geometrical text of Ibn Ṭahir al-Baghdādī [8, p.372–373] [4].

The structure of this problem is really different from the previous ones. It composed of a statement, a general algorithm, an example, a generalization with tacit conditions, computations and a proof by verification. Even if the algorithm

[2]In order to respect the editorial lines of this Report, we cannot illustrate our example by the text. So, we restrict our purpose to the main ideas

[3]See, for example, problems 23, 24 and 25 in the text of Ibn ᶜAbdūn [2].

[4]This problem can be read in an English version in [6, p.535–536]

given is totally explicit (no step seems to be tacit), it appears totally obscure and its reading is not sufficient to understand and generalize it. Indeed, several types of tacit knowledge reveal necessary. We will give three major features. First of all, the type of sharing is determined *a priori*. In his case, the diagram help us to understand the sharing. Secondly, the number of heirs is not the number of shares. But the last number is given by Islamic law which is not expose in the mathematical text. Here, we are in a case where 'tacit' is explicitly mentioned with special emphasis to social and religious knowledge. It remains, for us (present-day reader), obscure due to the lack of explanations. The last but not least, Ibn Ṭahir presents a general algorithm. Each step is detailed and even executed in order that the readers knows exactly computations to do. Each of them is explained by the general procedure, nevertheless the author doesn't indicate why this algorithm is correct. In particular, the author works on the number but not on the magnitudes they represent. The proof he writes is also restricted to check if the shares are equal. The correctness of the algorithm remains a tacit data in this context : everything is done as if the reader exactly knows that this algorithm is correct.

Conclusion

Reading geometrical texts from the *corpus* of misāḥa, I cannot agree with Polanyi's definition of 'Tacit Knowledge' quoted by the sociologist Collins in his *Tacit and Explicit Knowledge*, e.g. knowledge that cannot be made explicit, that cannot be expressed in words, sentences, numbers or formulas.

Indeed, in this paper, the examples mentionned show that a part of the work of the historian of mathematics, is precisely to *make the tacit explicit* however the 'tacit knowledge' can be defined.Indeed, in the case of our survey, knowledge is always be transmitted from person to person by books even if we can not ignore the eventual apprenticeship but we can control or modelize it several centuries after.

We don't have to forget also that the authors write their texts to be read by their contemporarians who share *habitus*, common education and so on. These authors cannot guess that their text will circulate in other regions (like the epistle of Ibn ^{c}Abdūn written in Andalus and found in a sub-Saharan library). They cannot guess either that it will be chosen to be translated in order to be used in another linguistic tradition (in the case of the book of Abū Bakr) or to be borrowed (directly or indirectly) as an obvious source by posterior mathematicians to produce their own text (Johannis of Muris and Fibonacci with the *Liber Mensurationum*). Thus, historian has only a selection of texts which is the result of an historical and social process. In this case, tacit knowledge is 'tacit' only keeping in mind that the sources that we have are incomplete. The local and oral traditions cannot be the only answer to characterize or elucidate this 'tacit' as it is often made.

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